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Figs. 1–19, March, 1957

BRAZOS RIVER BAR: A STUDY IN THE SIGNIFICANCE OF GRAIN SIZE PARAMETERS

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ABSTRACT

A bar on the Brazos River near Calvert, Texas, has been analyzed in order to determine the geologic meaning of certain grain size parameters and to study the behavior of the size fractions with transport. The bar consists of a strongly bimodal mixture of pebble gravel and medium to fine sand; there is a lack of material in the range of 0.5 to 2 mm, because the source does not supply particles of this size. The size distributions of the two modes, which were established in the parent deposits, are nearly invariant over the bar because the present environment of deposition only affects the relative proportions of the two modes, not the grain size properties of the modes themselves. Two proportions are most common; the sediment either contains no gravel or else contains about 60% gravel. Three sediment types with characteristic bedding features occur on the bar in constant stratigraphic order, with the coarsest at the base.

Statistical analysis of the data is based on a series of grain size parameters modified from those of Inman (1952) to provide a more detailed coverage of non-normal size curves. Unimodal sediments have nearly normal curves as defined by their skewness and kurtosis. Non-normal kurtosis and skewness values are held to be the identifying characteristics of bimodal sediments even where such modes are not evident in frequency curves. The relative proportions of each mode define a systematic series of changes in numerical properties; mean size, standard deviation and skewness are shown to be linked in a helical trend, which is believed to be applicable to many other sedimentary suites. The equations of the helix may be characteristic of certain environments. Kurtosis values show rhythmic pulsations along the helix and are diagnostic of two-generation sediments.

INTRODUCTION

Two of the most discussed yet most poorly understood topics in this day of quantitative geology are the concepts of grain size and sorting of sediments. Countless values have been published by research workers, recorded in innumerable theses, and secreted in oil company files; yet the meaning of all these figures and their ultimate geological significance (if any) are still quite obscure. One can hardly read a month's publications without encountering plots of sorting versus size or distance; contour maps showing values of grain size parameters; or statements of the alleged increase of sorting with sediment transport. Despite all this effort, few of these papers attempt to explain why or how the parameters are varying. If this vagueness is true of fairly simple ideas such as mean size or sorting, the situation with regard to more complex parameters like skewness or kurtosis is even worse. Little attempt has been made to relate these measures to the mode of deposition or to environmental characteristics, and most published papers simply tabulate these values without any evident attempt to understand or interpret them. One begins to wonder if all these lengthy computations are not wasted effort—do they show us anything of real value, or are they merely a deceptively impressive shell of figures surrounding a vacuum of geologic meaning?

A sand and gravel bar in the Brazos River near Hearne, Texas, was selected as a test case in which to determine, if possible, the geologic significance of such parameters as skewness and kurtosis in a localized environment; it was hoped that the facts learned here might aid in interpreting the meaning of these measures in other suites. A bar rich in gravel was chosen because it offered an opportunity to extend the size-versus-sorting trend into regions of essentially pure gravels. Hough (1942), Griffiths (1951), Inman (1949), Inman and Chamberlain (1955), and others have shown that the best sorting values are attained by medium to fine sands, and that sorting becomes worse as the sediments get either finer or coarser. Would the sorting continue to worsen as the trend was followed into the pure gravels, or would it reverse itself and begin to improve? This bimodal sand-gravel bar also afforded an opportunity to study the reason for the peculiar, almost universal lack of sedimentary particles in the range of 0.5 to 2 mm (Hough, 1942; Pettijohn, 1949). Depositional mechanism of the gravel and sand modes could be studied separately, their areal distribution and correlation with sedimentary structures could be obtained, and, in short, the complete size characteristics of a typical coarse river bar could be rigidly defined and interrelated so that some comparison might be made with other environments in future work.

LOCATION AND GROSS FEATURES OF THE BAR

The bar lies in a meander bend of the Brazos River about two miles north of Black Bridge, a crossing five miles west of Calvert, Robertson County, in central Texas (fig. 1). Steeply cliffed grassy terraces 10 to 25 feet above normal water level border the river here. Clement B. Thames, Jr. assisted the senior writer in surveying the bar with steel tape and Brunton compass in December, 1953. The deposit measured approximately 1100 feet long, and tapered from 300 feet wide upstream to 150 feet wide at the downstream end. Surface features were tied in to surveyed stake points by pace and compass. At the time of measurement the maximum bar height was 5 feet above river level; when revisited in December, 1955, it had undergone little change in form.

The bar, situated near mid-channel, consists basically of sandy pebble gravel with only a thin veneer of sand blanketing it mostly on the downstream side. The gravel foundation outcrops at the surface as a V-shaped area pointing downstream (fig. 1). The highest elevation is at the point of the "V," and each limb becomes lower upstream; the low area between the limbs of the "V" consists of gravel covered thinly with current-rippled sand. The thickest mass of sand coats the outer flanks of the gravel "V" and tails out downstream, but a few other small gravel patches also occur near that end of the bar. Thus the V-shaped gravel mound, which extends nearly all the way across the river, has controlled the accumulation of sand on its downstream side.

Extremely discoidal, well-rounded pebbles of Cretaceous limestone make up most of the gravel. On the bar surface pebbles are strikingly imbricated, dipping 20 to 30 degrees in the upcurrent direction. In detail, they are oriented in a complex pattern showing the path of water movement the last time the bar was submerged (fig. 1). Water apparently moves into the "trap" formed by the gravel "V," and spreads out over the limbs at a 45 degree angle to the trend of the channel. In the gravelly areas, the surface is covered with a clean lag deposit about one pebble thick resting on a smooth pavement where the gravel is packed tightly with interstitial sand. Where sand forms the surface of the bar, it is mostly current-rippled, but there are some areas of smooth sand and occasional areas thinly coated with a film of mud.

After the surface had been mapped, 27 sampling sites were distributed over the bar in an attempt to lay an approximately equi-spaced net and still sample all surface sediment types in representative proportions. Unless the water table was encountered, a hole 20 inches deep was dug in order to describe the stratigraphic section and sedimentary structures. From each site 2 samples were taken. A spot sample, designated "S," was collected over the interval from 1 inch to 2 inches below the surface and if the sediment was sand, a volume of about 2 cubic inches was procured. If any gravel was present, 8 to 10 cubic inches (all in the same 1″ to 2″ depth interval) was collected. The surface inch of sediment was not sampled, as in many instances it approached a lag deposit and might not be representative of the character of the sediment that eventually becomes buried. At each site a channel sample (designated "C") was also taken by cutting a sampling trench over the entire height of the hole. Special types of sediment, such as clean gravel layers present at some depth in the hole, were occasionally taken and designated by the code "X." These samples were separately treated in the statistical analysis of the results.

In digging sampling pits it soon became evident that the bar was made up of 3 sediment types which bore a constant stratigraphic relationship to each other. The sandy pebble gravels, containing over 30 percent (commonly 45 to 70 percent) gravel, occurred at the bottom of every hole that was dug deep enough, hence underlaid the entire bar in addition to outcropping over a large portion of its surface. These sediments showed a rude horizontal banding expressed by varying gravel content, but little cross-stratification was seen. Within this deposit were occasional sand layers and a few one- or two-inch thick streaks of "pure" openwork gravel (slightly coated with mud filtered down through the overlying sediment). It is interesting to observe the excellent imbrication of pebbles on the bar surface and to contrast it with the relative lack of imbrication of the buried pebbles. Perhaps imbrication takes place not on initial deposition of the pebbles and sand, but only when calmer currents later scour the surface and remove the interstitial sand—i.e. upon erosion of the sediment.

Above the foundation of sandy pebble gravel occurred a distinct layer of slightly gravelly medium sand usually containing a trace to 1 percent of granule-sized particles. This layer, at most 2 feet thick, consisted of sands which appeared to be clean and well sorted, forming tabular "torrentially" cross-laminated units 2 to 4 inches thick. Cross-laminae dipped about 30 degrees and were rather consistent in direction, swinging in a 50 degree arc on each side of the downcurrent direction.

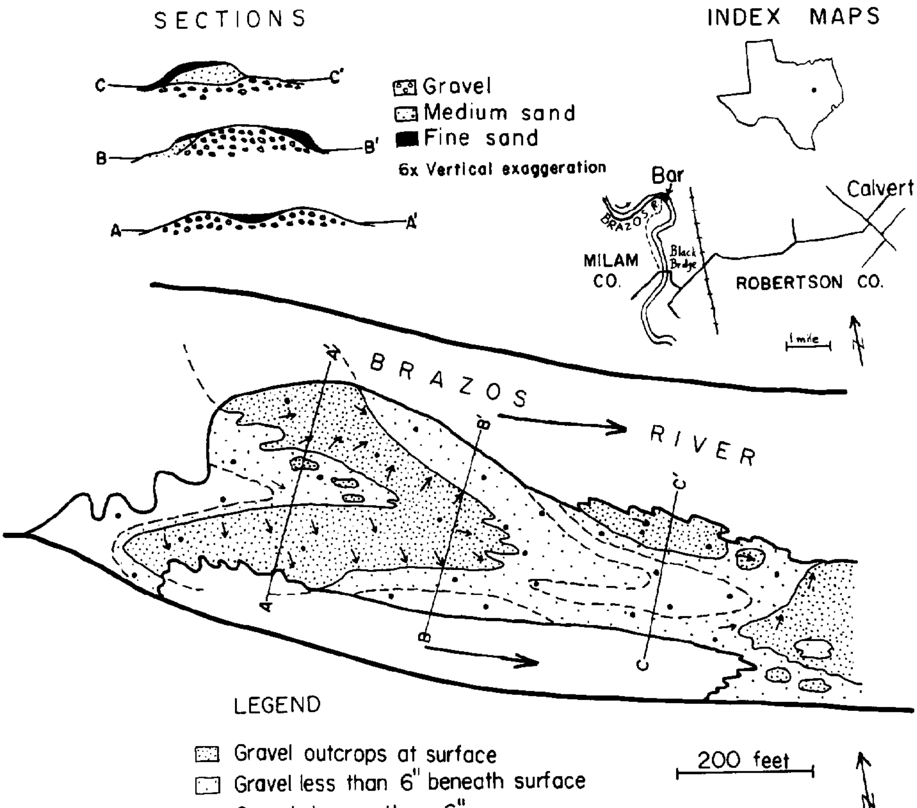

FIG. 1.—General features of the bar. Sandy pebble gravel forms the foundation of the bar, and outcrops as a V-shaped area pointing downstream.

The topmost layer of the bar consisted of fine sands and silty fine sands in a stratum at most 2 to 6 inches thick; occasionally this layer lay directly on the sandy gravel, and the middle layer was missing. This finest sediment type showed very small-scale lenticular cross-lamination with amplitudes of about 1 inch. Laminae were convexo-concave and had random dip direction.

In every hole the same sequence of layering was followed, and the layers were shaply bounded from each other with only rare interfingering. Thus the bar tends to become finer both upward stratigraphically and downstream.

GRAIN SIZE DISTRIBUTION

Laboratory Methods

If the sample consisted entirely of sand, it was split to between 50 and 75 grams, carefully disaggregated using a rubber cork and porcelain mortar, and sieved with Ro-Tap machine for 15 minutes. Eight-inch diameter Tyler screens, spaced at half-phi intervals (Krumbein, 1934), were used. Each fraction was weighed to 0.01 gm, and those amounts smaller than 1 gm were weighed to 0.001 gm. It is necessary to go to such accuracy when probability paper is used in plotting because the tails of the distribution are expanded so greatly. After being weighed, each fraction was examined with binocular microscope, and the percentage of aggregates (if any) was deducted from the raw weight as shown in the sample calculation (table 1). Deduction of aggregates is a critical step that is all too often overlooked in grain size analyses. Failure to do so has a very marked effect on sensitive measures such as skewness and kurtosis, which reflect the normality of the distribution. Few Brazos bar sieve fractions contained more than 5 percent aggregates, and most contained none. The corrected weights were cumulated, and the cumulative percentages were derived from the cumulated weights. This eliminates errors due to rounding off percentages and also insures that the anlysis will end at exactly 100.00 percent, a necessity when using probability graph paper.

TABLE 1.—*Method of computing analyses. The sample illustrated is not from this study but serves to illustrate the method*

Phi Interval	Raw Weight	Percent Aggregates	Corrected Weight	Cumulated Weight	Cumulated Percent
2.0–2.5	1.0	20	0.8	0.8	5.0
2.5–3.0	8.0	10	7.2	8.0	50.0
3.0–3.5	6.0	0	6.0	14.0	87.5
3.5–4.0	2.0	0	2.0	16.0	100.0

If the sediment contained any gravel at all, the entire sample was sieved through a 2 mm (-1ϕ) screen and the total amount of gravel retained on this screen was then sieved at an interval of one phi. The sand passing through the screen was split to a weight of between 50 and 75 grams, and this split was sieved as before. The weight of each sand fraction was then multiplied by the splitting factor (total weight of sand in the entire sample, divided by total weight of sand in the sieved split), and the weights cumulated together with the gravel portion of the analysis.

Cumulative percentages were then plotted against phi diameter on arithmetic probability paper. It is a waste of time to plot analyses on any other type paper (ordinary squared paper for example), as interpolation between data points is much more inaccurate and not reproducible. Values of skewness and kurtosis read off curves drawn on squared paper are worse than meaningless, because they depend almost entirely on the artistry of the draftsman and not on the sample characteristics. All curve parameters were read to the nearest 0.01ϕ, an accuracy which is meaningful when probability paper is used (Folk, 1955).

Bimodal Character of the Sediment

Brazos Bar sediments consist of a strongly bimodal mixture of pebble gravel with medium to fine sand. The gravel mode in most samples ranges between -2.0ϕ and -3.5ϕ (4 to 11 mm), while the sand mode generally lies between 1.2 and 2.8ϕ (0.45 to 0.15 mm) (fig. 4). In nearly all specimens, the minimum of the size distribution falls in the range of -0.5ϕ to $+0.35\phi$ (1.4 to 0.8 mm) (fig. 6). Only a few of the samples collected are unimodal.

As shown in figure 2, the relative proportion of the gravel and sand fractions varies widely between samples, but there are two preferred values: considering only spot samples (where only one sedimentation unit was collected), most samples contain 0.0 to 0.5 percent gravel, but another large group of samples contain 45 to 70 percent gravel. Between these two most frequent proportions occurs a pronounced gap, and indeed no spot samples collected had gravel contents between 8 and 30 percent. Channel samples show this same characteristic of having either almost no gravel or about 60 percent gravel, but the tendency is subdued. Special samples of clean, openwork gravel layers contained 90 to 95 percent gravel. This frequency distribution of gravel percentages, if it is characteristic of other sand and gravel bars, affirms the textural nomenclature proposed by Folk (1954) as the dividing lines between grain size classes occur at minima in the gravel frequency distribution.

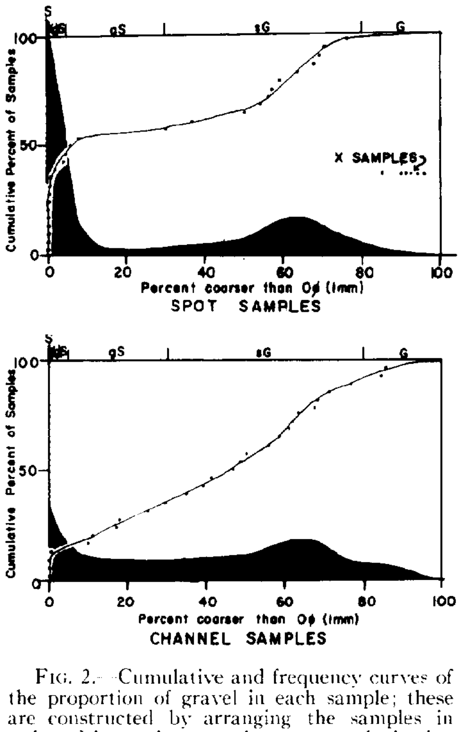

FIG. 2.—Cumulative and frequency curves of the proportion of gravel in each sample; these are constructed by arranging the samples in order of increasing gravel content and plotting them at equi-spaced intervals from 0% to 100%. In both channel and spot samples, there is a tendency for samples to have either 0% or about 60% gravel. Specially-collected layers of lag or openwork gravels contain 85 to 95% gravel (X samples). Textural designations are given at the top: S, sand; (g)S, slightly gravelly sand; gS, gravelly sand; sG, sandy gravel; and G, gravel.

In the Lafayette gravels in western Kentucky, Potter (1955) found the most common proportion to be 60 to 80 percent gravel, and Plumley (1948) found an average of 80 percent gravel in Black Hills terrace gravels. They have shown that because of restrictions of packing, a sediment with less than about 70 percent gravel must have had the sand and gravel deposited concurrently, i.e. the sand is not a later infiltration. Therefore, simultaneous deposition of the 2 modes must have occurred in the Brazos Bar, and infiltration of sand was unimportant as shown by the presence of openwork gravel layers.

The variation in percentage of gravel over the area of the bar is shown in figure 3, which is based on values obtained from channel samples only (spot samples show somewhat more fluctuation). Obviously this simulates figure 1 closely; the average gravel content of the top 20 inches of the bar is between 40 and 45 percent.

As will be shown later, the more complex grain size parameters such as mean size, standard deviation, skewness, and kurtosis are all rather close functions of the proportion of gravel in the samples. Consequently, maps showing the areal variation of these parameters are superfluous, and have not been included.

Character of the Gravel Fraction

The size characteristics of the gravel fraction may be analyzed independently of the "diluting" sand by taking the percentage of material coarser than 0ϕ (1 mm), dividing this percentage into 100 percent, and multiplying each of the gravel size grades by the resulting proportionality factor. In this way a cumulative curve for the gravel fraction alone may be plotted on probability paper. For example, if a sediment shows the following cumulative percentages: -4ϕ, 1%; -3ϕ, 7%; -2ϕ, 14%; -1ϕ, 18%; and 0ϕ, 20% all the cumulative percentages are multiplied by 100/20 and the curve is replotted as -4ϕ, 5%; -3ϕ, 35%; -2ϕ, 70%; and -1ϕ, 90%; and 0ϕ,

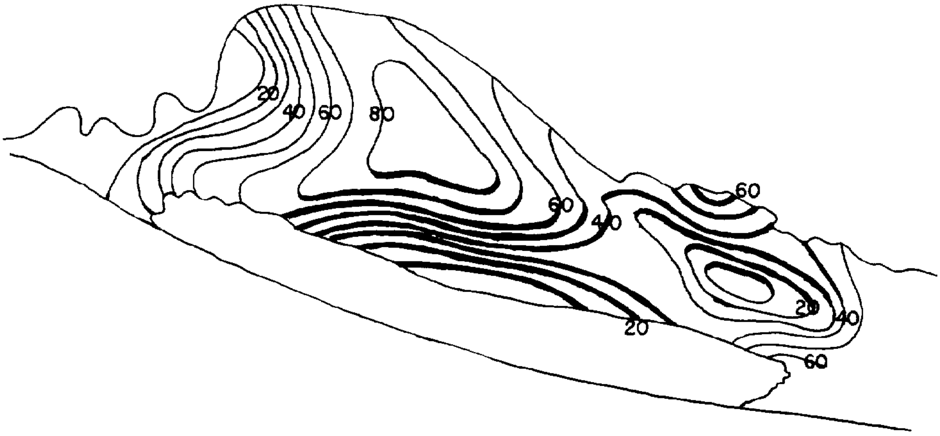

FIG. 3.—Contour diagram of the percent of gravel (material coarser than 0ϕ or 1 mm) in the top 20-inch section of the bar, based on channel samples.

100%. Actually of course, the Wentworth limit of gravel is -1ϕ, but in this particular sediment the significant size break occurs at 0ϕ so that the latter division is used throughout in calculations. Once a cumulative curve is plotted for the gravel fraction considered independently, it is possible to determine the mode, mean or median, standard deviation, skewness, and other critical properties for that fraction alone and to study how they vary in relation to position on the bar, percent gravel, size of the sand mode etc.

If only those samples with more than 10 percent gravel are considered, the grain size and sorting of the gravel fraction is surprisingly constant regardless of the proportion of gravel in the total sample. The average diameter of the gravel mode is -2.6ϕ (fig. 4) and two-thirds of the samples have gravel modes between -2.1ϕ and -3.4ϕ (4.5 to 10 mm). For each sample the sorting or standard deviation (σ_I, see later) of the gravel fraction alone was determined in the hope that it might prove an aid in characterizing river sediments if enough data are collected from other environments. The gravel fraction proved to be moderately to poorly sorted with σ_I averaging 1.1ϕ. Two-thirds of the samples had σ_I between 0.95 and 1.25; hence the sorting value is rather constant from sample to sample. All of the gravel fraction curves were nearly normal with skewness (Sk_I, see later) ranging from .00 to $+.15$ (equivalent $\alpha_3 = .00$ to $+.65$). The largest pebbles encountered had intermediate dimensions between 20 and 40 mm. The few samples of lag gravels collected from the surface of the bar and from the one- to two-inch layers of pure openwork gravel within the bar had modal sizes averaging -2.6ϕ with sorting values averaging 0.75ϕ and ranging from 0.5ϕ to 1.1ϕ; these were slightly better sorted than the sandy gravels discussed above. There is no correlation between the grain size of the

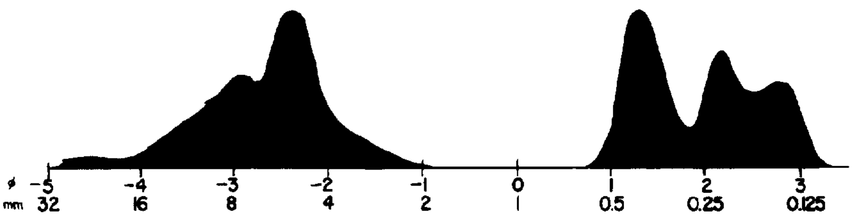

FIG. 4.—Frequency distribution of the sand and gravel modes for all samples.

gravel and sorting of the gravel within the limits of this study.

It is surprising to note that there is little or no correlation (r = +.05, negligible) between the diameter of the gravel mode and the percent of gravel in the sample. In other words, a sample with only 5 or 10 percent gravel has pebbles just as coarse as a sample consisting almost entirely of gravel (fig. 5). The fact that the gravel fraction has essentially constant size and sorting and that the pebble size is independent of the proportion of gravel in the sample indicates that the size distribution is chiefly a function of the grain size properties of the gravel supplied by the source in this particular area, and is but little affected by hydraulic factors or strength of the present depositing current. The size distribution of the gravel remains about the same whether the currents are strong (depositing little except gravel) or weak (depositing mostly sand with only a little gravel). Although an areal plot of percentage of gravel on the bar (fig. 3) correlates excellently with bar surface features, specifically the gravel "V," a similar areal plot of the modal size of the gravel showed only a random pattern.

Character of the Sand Fraction

If the sand modes for all samples are compiled into a frequency distribution (fig. 4), it is found that the most common modal diameters are 1.3ϕ, 2.2ϕ, and 2.8ϕ (.41, .22, and .15 mm). In relatively pure sands (containing less than 5 per cent gravel) the mode averages 2.5ϕ and ranges from 2.1ϕ to 3.0ϕ. The sand mode is one full Wentworth grade coarser in samples with 5 to 95 percent gravel, averaging 1.5ϕ and ranging from 1.1ϕ to 2.5ϕ (fig. 5). It is interesting to observe, however, that once the gravel content exceeds 5 percent, there is little or no correlation between the proportion of gravel and the size of the sand.

The average separation of the sand mode

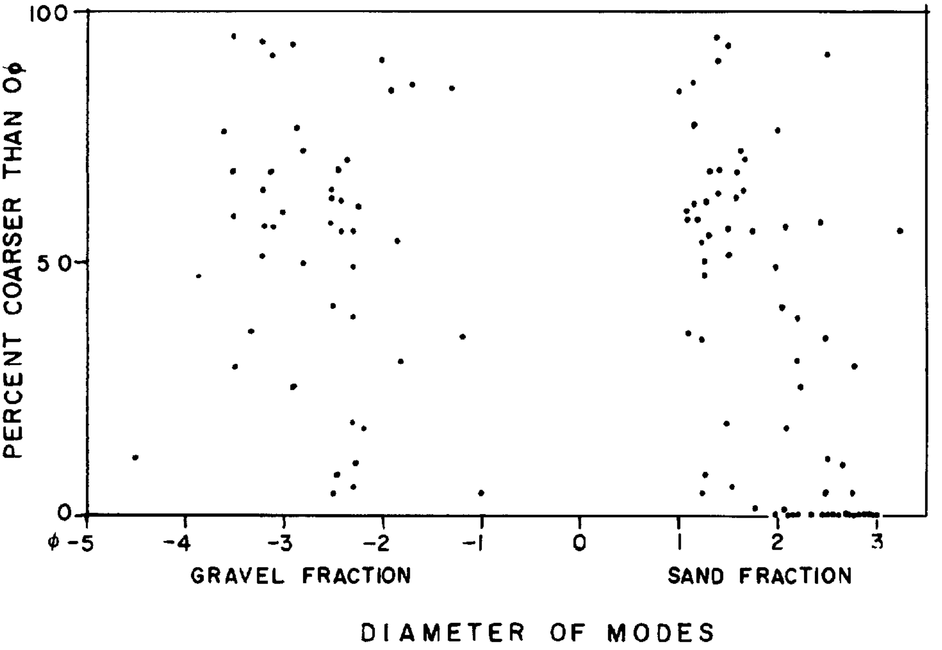

FIG. 5.—Modal diameter of the sand and gravel fractions as a function of the percent of gravel (coarser than 0ϕ) in the sample. Note narrow range of variation of both gravel and sand modes. There is no evident correlation between the amount of gravel and the size of the pebbles. Sands with no gravel have modes averaging 2.5ϕ; sands with 5 to 95% gravel have modes at about 1.5ϕ. This diagram illustrates that the size distribution of the modes themselves is chiefly a function of source area and is but little affected by strength of the depositing currents.

and the gravel mode in an individual sample (fig. 6) is 4.2ϕ, and two-thirds of the samples have separations in the range of 3.6ϕ to 5.1ϕ, or a millimeter ratio varying from about 21:1 to 35:1. Potter found an average separation of 4.4ϕ in the Lafayette Gravels. It is peculiar perhaps that there is no correlation between the size of the gravel mode and the size of the sand mode in the Brazos sediments.

MINERALOGY AND GRAIN SHAPE OF THE TWO MODES

In order to understand the cause of the bimodality, it is necessary to examine the mineralogy of the sand and gravel fractions. If sand grains were being produced by

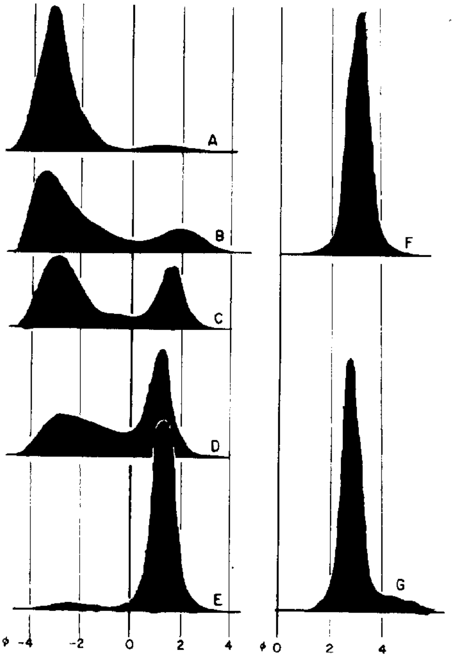

FIG. 6.—Representative frequency curves to cover the full range of variation in Brazos bar sediments. Note the nearly constant grain size of the modes, regardless of the variation in proportion of the modes. Each mode is within itself nearly symmetrical, although the total sample curves show wide variation in skewness and kurtosis. A is an openwork gravel layer; B, C, and D are typical sandy gravels. E is a slightly gravelly sand: F consists of the sand mode alone, and G is a silty sand. A, B, and G are positive-skewed, E and D are negatively skewed, and F and C are nearly symmetrical. A, E, and G are leptokurtic, C and D are platykurtic.

abrasion of the pebbles and some peculiarity of the abrasion process was responsible for the bimodality (such as pebbles of fine-grained granites disaggregating into their individual quartz and feldspar grains to produce the sand), then the sand and gravel should have similar composition. If the two modes were coming from different sources of supply, then their composition should differ.

Grain counts were made on three representative samples (fig. 7), and it was evident that the sand fraction was *not* originating through abrasion of the pebbles but was being reworked from older sand and sandstone formations which were not present as pebbles. In order of decreasing abundance the pebbles consisted of (1) very discoidal, well-rounded limestone; (2) subequant, subangular chert; and (3) subequant, round to subround vein quartz. As measured by sieving, limestone was largest with a mode of about -3ϕ (8 mm), virtually diappearing by 1ϕ. Likewise, vein quartz and chert with modes of -2.2ϕ (4.5 mm) almost vanished by 1ϕ. Limestone pebbles presumably were larger because of their low sphericity.

The sand grains were of two very distinct types. Over one-fifth of the grains were superbly rounded, lightly frosted quartz grains of extremely high sphericity, some almost perfect spheres. These quartz grains were probably inherited from Cretaceous supermature orthoquartzitic sands upstream and ranged in size from 0ϕ to 3.5ϕ. The other four-fifths of the sand grains, however, ranged from angular (in the 3ϕ size range) to at best subround (in the 0ϕ grade).

This mixture provides some interesting evidence concerning the classification of Dapples, Krumbein, and Sloss (1953) who have chosen as a measure of sediment maturity the percentage of well-rounded sand grains present in a deposit. This is an unfortunate choice because rounded grains may be so easily inherited (as they are in the Brazos Bar); it is not the most rounded grains but the most *angular* grains that are the true index of the amount of rounding taking place in the site of deposition. In a mixture of well-rounded and angular grains, the well-rounded grains are almost always reworked and have no bearing on the maturity of the latest sediment. Possibly something like the 16th percentile of the roundness distribution, rather than the mean, would be the best measure of the actual amount of rounding going on.

Chert and quartz show a peculiar reversal in roundness behavior. Above 0ϕ (1 mm) quartz is more rounded apparently because it is tough while the brittle chert pebbles tend to chip or split and remain subangular. Between 0ϕ and 1ϕ, both are of equal roundness, but in grains smaller than 1ϕ (0.5 mm) the chert grains are subangular with slightly though distinctly rounded edges while the quartz fragments are sharp and angular (excepting the readily recognizable inherited grains). Evidence of slight chert rounding was found in grains as small as 2.5ϕ. This seems to indicate that in sand-sized grains chert wears down and rounds faster than quartz because of slightly inferior hardness. A possible confirmation of this is found in the fact that chert grains are rare in supermature (well-rounded and well-sorted) orthoquartzites, while chert is quite common in sandstones of lower textural maturity (Folk, 1954) that have not suffered as much abrasion.

SUMMARY OF MODAL RELATIONSHIPS

If the proportion of gravel be taken as an index of current strength, then the following conclusions can be made:

1. In spot samples, the most common proportions of gravel are 0 percent and 60 percent: the least common proportion is 20 percent. At first, it might appear that this was due to the prevalence of two dominant levels of current strength. Rather, it is believed due to the fact that the sediment is bimodal and the size of the gravel is nearly constant. Thus, once a current is strong enough to move any gravel at all, it will move large quantities of it. If the sediment had a continuous range of particles from sand up to gravel size, then any percentage of gravel would be common; but in the Brazos Bar, there is a range of current strengths for which there are few available particles (-1ϕ to 1ϕ). Consequently the prominent percentages of gravel correspond to current strengths on each side of this gap.

2. On this bar, there is no correlation between current strength and grain size of the gravel. The diameter of the gravel mode appears to be nearly constant about a mean of -2.6ϕ.

3. On this bar, there is little correlation between current strength and the grain size of the sand mode, except that once the gravel content drops below 5 percent the sand is one phi unit finer. The sand mode averages 1.75ϕ.

4. Therefore, on this bar the size characteristics of the sand mode and of the gravel mode are controlled very largely by the source area and are little modified by stream action. The stream only affects the relative proportions of the two modes, not their sorting or grain size; hence its sorting effectiveness is very low. This may indicate that if sediments get finer downstream, it may be chiefly because the amount of gravel becomes less, rather than that its size is changing.

5. The correlation between sedimentary structures and size properties is tabulated below:

Sediment Type	Common Percent Gravel	Size of Sand Mode	Size of Grave Mode	Sedimentary Structures
a. Openwork gravels (rare streaks)	90–95	—	-1.8 to -3.4ϕ	none
b. Sandy gravels (bottom layer of bar)	45–70	1.5ϕ	-2.1 to -3.4ϕ	sub-horizontal banding
c. Slightly gravelly sands (middle layer of bar)	trace —5	2.2ϕ	—	consistent cross-bedding, 3″ amplitude
d. Fine sands (top layer of bar)	0	2.8ϕ	—	random cross-bedding, 1″ amplitude

STATISTISTICAL MEASURES USED IN THE ANALYSIS

In order to compare sedimentary environments with each other quantitatively, it is necessary to adopt precise measures of average size, sorting, and other frequency distribution properties. These properties may be determined either mathematically by the method of moments or graphically by reading selected percentiles off the cumulative curves (both methods summarized by Krumbein and Pettijohn, 1938). In this study the latter method was used, because it is much quicker and nearly as accurate. Sample-to-sample variation in any given property (say mean size) is so great that it is felt unnecessary to determine any single mean with extreme precision. For example, if one were assigned the task of obtaining the average height of 100 people, he would not have to go about it by measuring each person to the nearest 0.001″.

In a suite of samples as strongly bimodal as those found on the Brazos River Bar, most of the grain size frequency curves are very non-normal. Therefore the commonly-used graphic measures of mean size, sorting and other statistical parameters are inadequate, because they are based on only two or three points read off the cumulative curve; strongly non-normal curves require more detailed coverage in order to render their properties accurately. Consequently the writers have been forced to use a new series of statistical measures corresponding closely to those suggested by Inman (1952), but including more points on the curve. For nearly normal curves the two systems give almost identical results, but for skewed and bimodal curves the system introduced here is superior. In the interests of consistency, the new measures are now used for all size analyses regardless of their modality.

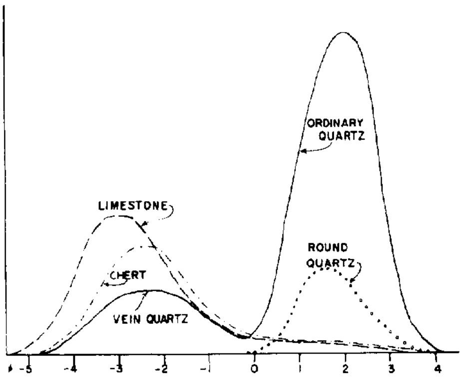

FIG. 7.—Lithologic and mineralogic size-frequency curves, representing a weighted composite of three Brazos bar samples. The sand is not originating by breakdown of the pebbles, but is being reworked from older sand and sandstone deposits. Note nearly symmetrical frequency curves for each constituent taken individually.

Mean Size.—Inman (1952) suggested $(\phi16+\phi84)/2$ as a measure of mean size. This serves quite well for nearly normal curves, but fails to reflect accurately the mean size of bimodal and strongly skewed curves. Therefore we have used another measure of the mean, M_z, determined by the formula

$$M_z=\frac{\phi16+\phi50+\phi84}{3}.$$

Here, the $\phi16$ may be considered roughly as the average size[1] of the coarsest third of the sample, and the $\phi84$ as the average size of the finest third; the addition of the $\phi50$ (the average of the middle third) thus completes the picture and gives a better overall representation of the true phi mean. To compare the accuracy of the two graphic systems, ten size analyses were chosen to represent the full range of textures in the Brazos Bar, and the mean size was computed for each sample by the method of moments (Krumbein and Pettijohn, 1938). The deviation of the graphic mean from the moment mean was then determined for both systems. In a range of means from -2.0ϕ to $+3.5\phi$ (in which all curves but two were strongly bimodal), the root mean square (rms) deviation[2] of $M\phi$ (Inman) from the moment mean was 0.25ϕ (maximum observed deviation 0.56ϕ) while the rms deviation of M_z (Folk and Ward) was only 0.12ϕ (maximum observed deviation 0.22ϕ); thus M_z gives twice as accurate an approximation to the moment mean.

Mode.—No good mathematical formula exists for accurate determination of the

[1] More precisely, the *median* of the coarsest third, etc.

[2] The root mean square (rms) deviation is computed by taking the deviations of $M\phi$ (or M_z) from the moment mean, squaring them, summing the squares, dividing the sum of the squares by the number of values, and taking the square root of this quotient. Approximately two-thirds of the individual deviations will then be less than the value of the rms deviation. A simpler but less useful measure is the mean deviation (sum of the deviations divided by the number of deviations, i.e. the average deviation), which is 0.09ϕ for M_z and 0.20ϕ for $M\phi$.

mode. The best approximation is probably that given by Croxton and Cowden (1939, p. 213), but this works well only when the distribution is symmetrical in the region neighboring the mode and fails in skewed curves. The writers have here used a repetitive trial-and-error method, wherein the percentage of sample actually occurring within a size interval of 0.5 phi unit is read directly from the cumulative curve. Readings are taken at successive steps of 0.1ϕ (e.g. first 1.2ϕ to 1.7ϕ, next 1.3ϕ to 1.8ϕ, etc.) until a maximum percentage is reached. The maximum percentage occurring within a half-phi diameter range in any sample has been given the term "modal concentration" and may have some value as an auxiliary measure of the degree of sorting in the region about the mode.

Median.—In the opinion of these writers, the median is a very misleading value and should be abandoned as a measure of average size inasmuch as it is based on only one point of the cumulative curve. For example a sediment consisting of 40 percent pebbles and 60 percent fine sand may have the same median as one with 60 percent fine sand and 40 percent clay.

Standard Deviation.—As a measure of sorting, Inman (1952) followed Krumbein (1938) and Otto (1939) and suggested the phi standard deviation,

$$\sigma_\phi = \frac{\phi 84 - \phi 16}{2},$$

thus using a uniformity measure similar to that employed by statisticians. For many normal curves this measure is adequate; however, it is based only on the central part of the distribution and ignores fully one-third of the sample—specifically, the "tails," which offer some of the most valuable information. Thus a sand with 10 percent pebbles and 10 percent clay may turn out to have a sorting value the same as pure sand. For complex distributions like the Brazos River bar (or, as matter of fact, for many neritic sediments where small amounts of clay are mixed with a dominant sand fraction), this parameter gives misleadingly high sorting values. The remedy is simple: include more of the distribution curve in the sorting measure. Although it would be theoretically best to include everything from the first to the 99th percentiles, Inman (1952) has shown that data are seldom reliable beyond the 5th and 95th percentiles. Hence these percentiles provide a practical end point, and if they are used only one-tenth of the sediment is excluded from the sorting measure. Inasmuch as the spread between the 5th and 95th percentiles includes 3.3 standard deviations, a standard deviation measure based only on the extremes,

$$\sigma_\phi = \frac{\phi 95 - \phi 5}{3.3},$$

could be developed. But neither the $\phi 84 - \phi 16$ measure nor the $\phi 95 - \phi 5$ measure is adequate by itself for complex bimodal sediments, and a superior over-all measure of sorting could be obtained by combining the two and taking their average. This measure, called the Inclusive Graphic Standard Deviation, is found by the formula[3]

$$\sigma_I = \frac{\phi 84 - \phi 16}{4} + \frac{\phi 95 - \phi 5}{6.6}.$$

Again to compare the relative accuracy of the two graphic systems, the standard deviation was computed by the method of moments for the same 10 samples discussed above; these showed a range of σ_I from 0.40ϕ to 2.60ϕ. For σ_ϕ (Inman) the rms deviation was 0.31ϕ, maximum 0.55ϕ; for σ_I (Folk and Ward) the rms deviation was 0.18ϕ, maximum deviation 0.32ϕ. Thus σ_I gives a considerably more accurate approximation to the moment σ.

In discussing sorting, it is convenient to have a verbal scale, particularly so that information may be communicated to non-specialists. Plotting of hundreds of analyses from many different environments has suggested the following divisional points: σ_I under 0.35, very well sorted; σ_I 0.35–0.50, well sorted; σ_I 0.50–1.00, moderately sorted; σ_I1.00–2.00, poorly sorted; σ_I 2.00–4.00, very poorly sorted; σ_I over 4.00 extremely poorly sorted. With the exception of the lowest limit the scale is geometric with a

[3] Many analyses of clayey sands and muds never attain the 84th or 95th percentiles. For these we have adopted the convention of extrapolating from the last point determined by pipette or hydrometer to 100% at 14ϕ using a straight-line plot on arithmetic paper. Intercepts are then read off the extrapolated curve.

ratio of 2. The smallest σ_I value so far encountered in our analyses is 0.20, while some sorting values as poor as 8.0 or more have been determined.

It may be argued that any attempt to set verbal limits on sorting values is foolish, because as shown by many (Inman, 1949; Griffiths, 1951), sorting is a rather closely-controlled V-shaped or sinusoidal function of mean size; hence about the only sediments falling in the "well-sorted" category would be the medium and fine sands, and all clays, silts, and most gravels would be poorly sorted to very poorly sorted. The frequent generalization that sorting increases with transport is in many suites simply due to the fact that the mean size of a sediment changes with transport, and the improvement in sorting is dependent only on the decreasing mean size, not the distance. As Inman (1949) suggested, once the sediment attains a minimum σ (best sorting), if it continues to get finer it will "round the turn" on the curve and sorting will worsen with further transport. A truly meaningful verbal scale of sorting will be developed only when the general trend of the size versus sorting relationship is worked out for a great number of environments. One will then be able to say, for example, that his sediment has a σ_I 0.25ϕ lower than the average sediment of that same mean grain size.

Skewness.—Inman suggested two measures of skewness: one,

$$\alpha_\phi = \frac{\phi 84 + \phi 16 - 2\phi 50}{\phi 84 - \phi 16},$$

to determine the asymmetry of the central part of the distribution and the other,

$$\alpha_{2\phi} = \frac{\phi 95 + \phi 5 - 2\phi 50}{\phi 84 - \phi 16},$$

to measure the asymmetry of the extremes. Again, a better measure of *over-all* skewness may be obtained by averaging[4] these two values by the formula, Inclusive Graphic Skewness

$$Sk_I = \frac{\phi 16 + \phi 84 - 2\phi 50}{2(\phi 84 - \phi 16)} + \frac{\phi 5 + \phi 95 - 2\phi 50}{2(\phi 95 - \phi 5)}.$$

Using this measure (as in Inman's original formula) skewness is geometrically independent of sorting, perfectly symmetrical curves have $Sk_I = .00$, and the absolute mathematical limits are -1.00 to $+1.00$; however, very few curves have Sk_I beyond $-.80$ or $+.80$. Positive values of Sk_I indicate that the samples have a "tail" of fines; negative values indicate a tail of coarser grains. Plotting of many grain size analyses has suggested the following verbal limits: Sk_I -1.00 to $-.30$, very negative-skewed; Sk_I $-.30$ to $-.10$, negative-skewed; Sk_I $-.10$ to $+.10$, nearly symmetrical; Sk_I $+.10$ to $+.30$, positive-skewed; and Sk_I $+.30$ to $+.100$, very positive-skewed. Plotting Sk_I against the value of skewness α_3 derived from the method of moments (Krumbein and Pettijohn, 1938) reveals that Sk_I equals approximately 0.23 α_3 and α_3 equals about 4.35 Sk_I.

Kurtosis.—Kurtosis, as used by most sedimentationists, measures the ratio of the sorting in the extremes of the distribution compared with the sorting in the central part and as such is a sensitive and valuable test of the normality of a distribution. Many curves designated as "normal" by the skewness measure turn out to be markedly non-normal when the kurtosis is computed. The Graphic Kurtosis (K_G) used here is given by the formula

$$K_G = \frac{\phi 95 - \phi 5}{2.44(\phi 75 - \phi 25)}.$$

In a normal Gaussian curve, the spread in phi units between the 5th and 95th percentiles should be 2.44 times the spread between the 25th and 75th percentiles. Thus, using the equation here, *normal* curves have $K_G = 1.00$. A curve with $K_G = 2.00$ is leptokurtic or excessively peaked (relatively better sorted in the central area than in the tails), inasmuch as the $\phi 5$ to $\phi 95$ spread is exactly 2.00 times as large as it should be for a given $\phi 25$ to $\phi 75$ interval. If $K_G = 0.70$ (platykurtic or deficiently peaked), the $\phi 5$ to $\phi 95$ spread is only 0.70 of what it should be in a normal curve with the same $\phi 25$

[4] In Inman's original equation (1952, p. 137) the denominator of the second term above was σ or $(\phi 84 - \phi 16)/2$. Using this denominator, it is possible to get skewness of absolute value greater than 1.00 in strongly leptokurtic and asymmetrical curves, and skewness becomes to some extent a geometric function of kurtosis. A geometrically independent measure is retained if we use $(\phi 95 - \phi 5)$ in the denominator.

to $\phi 75$ interval. The advantage of the kurtosis measure introduced here over previous measures lies in its simple relation to the normal curve which has $K_G = 1.00$; also the geometric significance can be easily visualized.

Based on analysis of hundreds of samples, the following verbal limits have been used for the kurtosis measure: K_G under 0.67, very platykurtic; K_G 0.67 − 0.90, platykurtic; K_G 0.90 − 1.11, mesokurtic; K_G 1.11 − 1.50, leptokurtic; K_G 1.50 − 3.00 very leptokurtic; and K_G over 3.00, extremely leptokurtic. In this scale the lower kurtosis limits are the reciprocals of the higher ones. The absolute mathematical minimum for the measure is 0.41, but no samples yet analyzed have had K_G below 0.50. There is no theoretical maximum for the measure,

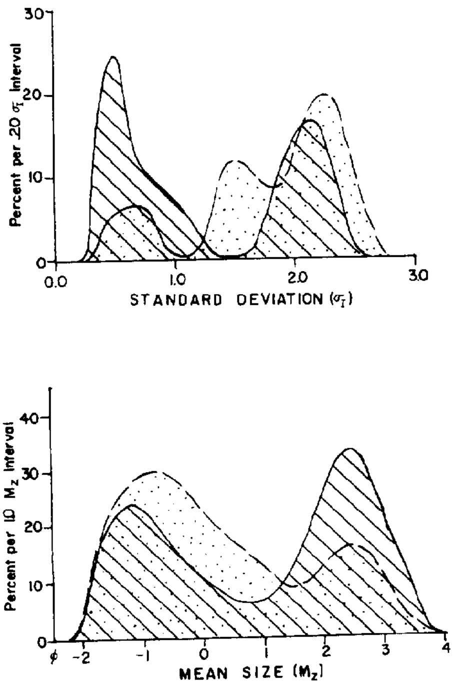

FIG. 8.—Frequency distribution of standard deviation values and mean size values for Brazos bar sediments. Spot samples are shown in a diagonal-line pattern, channel samples by dot pattern. The ordinate gives the percentage of the analyzed samples falling in the given interval (e.g. the curve for standard deviation of channel samples is 11% at $\sigma_I = 1.5$; this means that 11% of the samples had σ_I values between 1.4 and 1.6).

but $K_G = 8.0$ appears to be about the highest value attained in natural sediments.

It is evident that the distribution of K_G is itself strongly non-normal, since natural sediments average around $K_G = 1.00$ with a range from 0.50 to 8.0. Hence for plotting graphs and for statistical analyses the distribution has been approximately normalized by using the transformation $K_G' = K_G/(K_G + 1)$. Normal Gaussian curves then have $K_G' = 0.50$ ($K_G = 1.00$), and the range of K_G' in natural sediments is about 0.33 to 0.90.

FREQUENCY DISTRIBUTION OF PARAMETERS IN THE BRAZOS BAR

It is of considerable theoretical interest to examine the frequency distributions of the values of the 4 size parameters obtained from the Brazos Bar samples, as this may be one of the best ways in which to characterize environments. For example, the 54 channel and spot samples have 54 different skewness values. These questions arise: what is the average skewness of these samples; what is the standard deviation of the skewness values (i.e. how wide a range of skewness is shown by the central two-thirds of the samples); is the skewness distribution unimodal or bimodal; and one may even consider such problems as the skewness of the frequency distribution of skewness values or the kurtosis of the skewness frequency distribution. In this way any size parameter (such as skewness, sorting, etc.) may be treated in exactly the same way as any other numerical parameter obtained for the 54 samples (say porosity, feldspar content, or sphericity), and its frequency distribution may be analyzed in similar fashion.

Mean Size.—Mean size (M_z) ranged from -1.7ϕ to $+3.2\phi$ (3.3 to 0.11 mm) for spot and channel samples (fig. 8), although some of the specially chosen X samples (clean gravel layers) had M_z from -2.5ϕ to -3.3ϕ (5.6 to 10 mm). Spot samples gave an extremely bimodal distribution with a mean M_z of 1.1ϕ and a standard deviation of 1.7ϕ (in other words if the distribution of M_z were normal, about two-thirds of the M_z values would fall between 2.8ϕ and -0.6ϕ). In such a non-normal distribution, however, these values have little significance, and it is more meaningful to say that the largest clustering of M_z values is between 1.8ϕ and 3.2ϕ, representing the relatively pure fine sands, with a somewhat smaller cluster about -1.5ϕ to 0.0ϕ (sandy gravels). Few spot samples have M_z between 0.5ϕ and 1.5ϕ. Channel samples show a similarly bimodal distribution of M_z, but the maximum concentration is at -1.5ϕ to -1.0ϕ.

Standard Deviation.—Standard deviation (σ_I) values ranged from 0.40ϕ to 2.58ϕ on the Brazos Bar (fig. 8). Spot samples had a mean standard deviation of about 1.2ϕ, but this value is without significance because the distribution of standard deviations is very markedly bimodal. There is a great clustering of σ_I values at 0.40ϕ to 0.50ϕ, representing the well- to moderately-sorted "pure" sands, and an almost equally large group at $\sigma_I = 1.80\phi - 2.30\phi$, representing the poorly to very poorly sorted sandy gravels. No σ_I values between 1.20ϕ and 1.80ϕ occurred.

Channel samples had a mean σ_I value of about 1.8ϕ, but the greatest clustering occurred about a value of 2.0ϕ–2.4ϕ, with another very minor group at 0.40ϕ–0.80ϕ. Again the distribution of values was nonnormal. There was little difference in sorting between spot and channel samples, except where the spot samples consisted of sand alone and the channel sample at the same locality contained some gravelly layers deeper in the test pit.

Preliminary work at the University of Texas has shown that most Texas beach sands have σ_I in the range of 0.20–0.40; yet the best sorted Brazos Bar sands, with approximately the same mean grain size as the beach sands, have σ_I ranging from 0.40–0.60ϕ. Hence, the beach sands are almost twice as well sorted. This may be due in part to the method of deposition in the two environments. The Brazos bar sands are deposited as continuous cross-bedded units, in which sediment is rather rapidly dumped down the front of the advancing sand mass as foreset beds. There is little opportunity for sorting, as the sediment, once deposited, is buried rapidly and no gentle reworking takes place. On the contrary, the beach sands tend to be raked back and forth by the continual motion of wave swash, which produces laminae nearly parallel with the slope of the beach. The continuous reworking apparently results in good sorting.

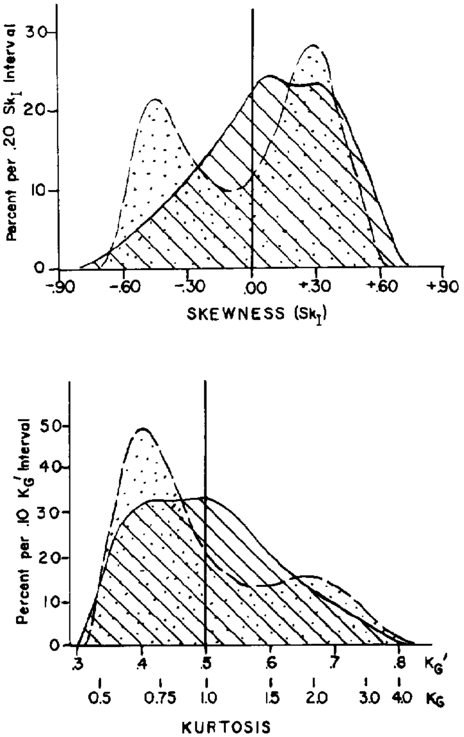

FIG. 9.—Frequency distribution of skewness and kurtosis values from Brazos bar sediments. Spot samples are shown in a diagonal line pattern, channel samples in a dot pattern.

Skewness.—Skewness values ranged widely from $-.68$ to $+.53$ (fig. 9). Spot samples showed a nearly normal distribution of Sk_I grouped about a mean skewness of $+.09$ with a standard deviation of 0.34 (i.e. two-thirds of the samples had skewness values ranging between $-.25$ and $+.42$). Thus most of the samples have a tail to the right, i.e., an excess of fine material for a normal curve.

Channel samples, on the other hand, had distinctly bimodal distribution of skewness values, with a large clustering about $Sk_I = +.20$ to $+.40$ (sandy gravels with a tail in the sand sizes) and another somewhat smaller grouping at $-.30$ to $-.55$ (gravelly sands, dominantly sand with a small tail of gravel). Few skewness values occurred in the range of .00 to $-.20$.

Kurtosis.—In analyzing the data, the transformed value K_G' has been used instead of the actual K_G values. Plotted on this scale (fig. 9), the distribution of kurtosis values is nearly normal in spot samples with a mean K_G' of 0.49 (corresponding to $K_G = 0.96$) and a standard deviation of 0.12 (two thirds of the K_G' values fall in the range from 0.39–0.62, and corresponding K_G values from 0.64 to 1.63). The range of K_G' was from 0.35 to 0.74 (K_G from 0.54–2.85), and platykurtic and leptokurtic samples occurred with about equal frequency.

Channel samples showed a non-normal distribution of kurtosis with a large clustering about $K_G' = 0.36–0.47$ (platykurtic—$K_G = 0.56–0.89$), and another much smaller grouping at $K_G' = 0.62–0.70$ (leptokurtic, $K_G = 1.63–2.33$).

INTERRELATION OF THE FOUR SIZE PARAMETERS

To understand the geological significance of the four size parameters, it is necessary to plot them against each other in turn as scatter diagrams. In this way their interrelationships are revealed, and a wealth of meaning comes to light. Although in theory the measures are geometrically independent, in actual practice it is usually found that for a given suite of samples the measures are linked by some mathematical relationship. Perhaps the relationships and trends may be clues to the mode of deposition and will add one more criterion for identifying environments by size analyses.

First, all 6 two-variable scatter plots are discussed, and next it is shown that all 4 parameters of the frequency distribution can be combined in a helical trend. The geological significance of this helix is then interpreted, and its application to other sedimentary suites is discussed.

Mean Size versus Standard Deviation.—Generally plots of this type give a great amount of information about an environment (Inman, 1949, p. 64). If a wide range of grain sizes (gravel to clay) is present, scatter bands often form some segment of a broadened M-shaped trend. Often only a V-shaped or inverted V-shaped trend develops if the size range is smaller, and if the range is very small, only one limb of the V may occur. Minima of best sorting coincide with prominent modes in the sediment, and maxima (poorest sorting) correspond to mean sizes midway between modal diameters. In the Brazos bar, an inverted V-shaped trend occurs with a suggestion of a slight upward hook on the right limb (fig. 10). The rare clean gravel layers with M_z about -3ϕ are not too badly sorted with σ_I about 1.0. As the pure and essentially invariant gravel mode becomes mixed with more and more sand, the mean size decreases and the sorting worsens until the highest σ_I values are attained when the sediment consists of subequal proportions of sand and gravel. These samples have a mean size about half-way between that of the sand mode and the gravel mode, i.e. about -1ϕ to 0ϕ and sorting values of $1.75–2.5\phi$.

As the sand mode increases in abundance and the gravel diminishes, the mean size becomes finer and the sorting begins to improve. Finally, in those samples consisting only of the "pure" sand mode, best sorting occurs at a mean size of $2.1\phi–2.7\phi$ with σ_I about $0.40\phi–0.60\phi$. Having about half as large a standard deviation, the pure sand mode is thus inherently better sorted than the pure gravel mode. This apparently is the result of the type of material supplied by the source area, as the present river deposition has little effect on the sorting of the individual modes.

The suggestion of an upward hook at the right of the diagram is caused by mixture of the dominant sand mode with a small

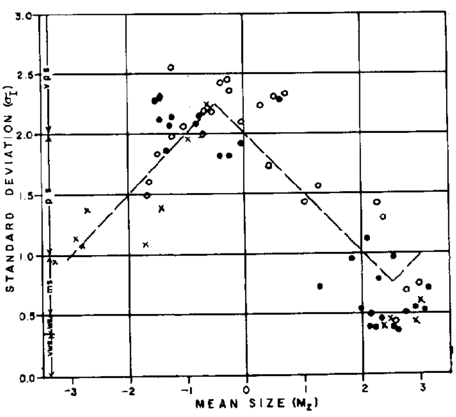

FIG. 10.—Scatter plot of mean size versus standard deviation (sorting). Spot samples shown by filled circles, channel samples by open circles, and special samples by X. Letters along the left margin give verbal limits on sorting: vws, very well sorted; ws, well sorted; ms, moderately sorted; ps, poorly sorted; vps, very poorly sorted. Trend line is discussed in the text.

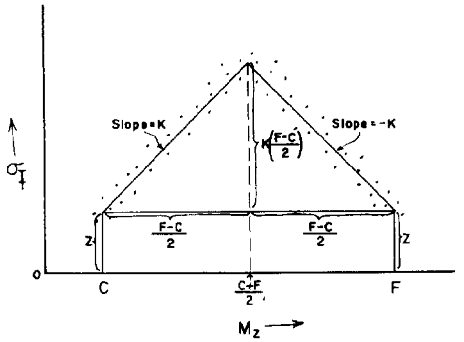

FIG. 11.—Geometry of the general equation for sorting in bimodal sediments, derived in the text. C and F are the phi diameters of the coarse and fine modes, respectively; Z as the σ_I value of the modes when present alone. Poorest sorting occurs at a mean size midway between the sizes of the two modes, i.e., at the position (C+F)/2. The trend followed by a series of samples is shown diagramatically by the dots.

amount of a third mode in the silt sizes. This further decreases the mean size and starts to worsen the sorting again, by the same mechanism of adding two modal distributions together. Presumably if enough fine samples had been taken, the plot would have developed another inverted V-shaped trend to the right of the present one, with a maximum σ_I corresponding to a subequal mixture of the sand mode and the silt mode and a minimum σ_I corresponding to the size of the pure silt mode.

As shown later, the true trend is probably a sine curve repeated over several wave lengths, each minimum corresponding to a mode in the suite of sediments being considered. However, an equation nearly as adequate in predicting sorting values may be developed under the assumption that the trend is composed of two intersecting straight-line segments. This equation is $\sigma_I = 2.25\phi - 0.5\,|M_z + 0.5\phi| \pm 0.4\phi$, where M_z is the mean size of any given sample and σ_I is the corresponding sorting value. The expression $|M_z + 0.5\phi|$ stands for the absolute sum of M_z plus 0.5ϕ taken with sign ignored; i.e. if a given sample has an M_z of -2.0ϕ, the value of the term is $-0.5\,|-2.0\phi + 0.5\phi| = 0.5\,|-1.5\phi| = -0.5 \times 1.5\phi = -.75\phi$. The term $\pm 0.4\phi$ is the standard error of estimate and signifies that about two-thirds of the σ_I values will fall within the range of the predicted σ_I value $\pm 0.4\phi$.

This trend line suggests that a general equation for sorting in bimodal sediments can be evolved. The first term of the equation (in the example here, 2.25ϕ) is the value of σ_I at the apex or poorest sorting point of the inverted V trend. The multiplicand of the second term is a constant which is fixed by the slope of the trend lines (assuming that they are of equal slope, sign ignored), and gives the change in σ_I per unit change in M_z (here it is 0.5). In the expression $|M_z + 0.5\phi|$, which in reality is $|M_z - (-0.5\phi)|$, the second term represents the mean size value at which poorest sorting takes place. In most sediments this appears to be at a mean size halfway between the sizes of the two modes. If the slope constant is designated K, the phi diameter of the coarser mode is called C, the phi diameter of the finer mode is called F, and the sorting value of the pure sand mode and pure gravel mode when present separately[5] is Z, then the first term in the generalized equation (corresponding to 2.25 in the specific example here) will be $K(F-C)/2+Z$ (fig. 11). The mean size value at which poorest sorting takes place, being halfway between the two modes, can be represented by their average $(F+C)/2$; therefore the second term in the generalized equation becomes

$$K\left|M_z - \frac{F+C}{2}\right|,$$

and the final equation is

$$\sigma_I = K\left(\frac{F-C}{2}\right) + Z - K\left|M_z - \frac{F+C}{2}\right|. \qquad (1)$$

In sediments deposited under conditions similar to the Brazos River bar where $K=0.5$ and $Z=0.75$, the equation reduces to

$$\sigma_I = \frac{F-C}{4} + 0.75 - 0.5\left|M_z - \frac{F+C}{2}\right|. \qquad (2)$$

Geologically, the F and C terms reflect the modal size of the material contributed by the source area or areas; Z represents the individual sorting of these fractions, strongly affected by the character of the source; and K represents the interplay between two factors: (1) distinctness of the modes contributed by the source areas, and (2) efficacy of the transporting agent in doing its own

[5] This equation applies only if the sorting of the coarser mode and the finer mode are nearly equal.

sorting on the material contributed to it. If the modes contributed by the source of supply are only two, are well sorted and essentially invariant, and are of widely differing size, then K will approach 1.0. If there are many indistinct modal distributions contributed by the source or if these modes are in themselves widely variant or poorly sorted, then K may approach 0, and there will be no significant change in sorting with change in size. Secondly, if the environment of final deposition is very ineffective in sorting, K will remain high. If the environment is effective in sorting, then K will be lowered because not only will the modal sediments be well-sorted, but the hybrid sediments consisting of mixtures of modes, instead of having the poor sorting normally associated with these mixtures, will have good sorting and the inverted V trend will be flattened.

Hence K is high, for example, in some lagoonal sediments, because a relatively nonvariant, well-sorted sand mode is produced on the beaches and a non-variant clay mode accumulates in the lagoon. When these two sediments become mixed and deposited in the low-energy environment of the lagoon where no further sorting takes place, then K will remain at a maximum. K should be at a minimum for beach sands fronting a coast with many rivers contributing sediment from different types of source areas. Here sediments entering the area are polymodal, the modes are blurred, and the environment is one in which efficient sorting is going on. Thus sediments of all grain sizes can become well sorted, and there should be little association of sorting with size.

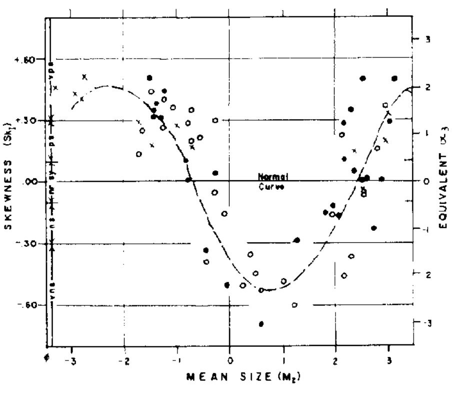

FIG. 12.—Scatter plot of skewness versus mean size. Spot samples shown by filled circles, channel samples by open circles, and special samples by X. The trend is markedly sinusoidal, with nearly equal numbers of positive-skewed and negative-skewed samples. Letters along the left margin give verbal limits for skewness; vns, very negative-skewed; ns, negative-skewed; nr sy, near-symmetrical; ps, positive-skewed; vps, very positive skewed. Equivalent α_3, based on the method of moments, is shown along the right margin.

Mean Size versus Skewness.—In this bimodal sediment, skewness is very closely a function of grain size (fig. 12). The trend is markedly sinusoidal. The pure sand mode when it occurs by itself (at a mean size of about 2.5ϕ) produces a symmetrical size curve, but the addition of increasing quantities of gravel mode[6] imparts negative skewness, which becomes most extreme at a mean size of $+0.7\phi$, where the skewness averages $-.50$ or more. As more gravel is added and the two modes become equal in quantity, the trend reverses itself and sweeps briefly through a region of symmetry ($Sk_I = .00$ at about $M_z = -0.5\phi$). As the amount of gravel comes to exceed the amount of sand, the size curves become more and more positive-skewed reaching a maximum skewness of $+.50$ at $M_z = -2.2\phi$. In the pure gravels the skewness decreases, but in the samples collected here Sk_I never attains .00 (theoretically Sk_I should reach .00 again in a pure openwork gravel mode, but these samples always contained 5 percent or more sand which imparted a positive skewness).

In sediments finer than 2.5ϕ, the decrease in size comes about through addition of small quantities of silt mode to the dominant sand mode. This explains the positive skewness values at the right edge of the trend. Thus, to generalize, the pure modal fractions are in themselves nearly symmetrical, but the mixing of the two modes produces negative skewness if the finer mode is most abundant and positive skewness if the coarser mode is most abundant. An equal quantity of the two modes results in a symmetrical curve.

Mean Size versus Kurtosis.—This relationship is complex. The curve shown in figure 13 is theoretical, based on what the trend should look like given the two modes present in the Brazos bar and mixing them in vari-

[6] Sk_I is affected as soon as the gravel content exceeds 5%.

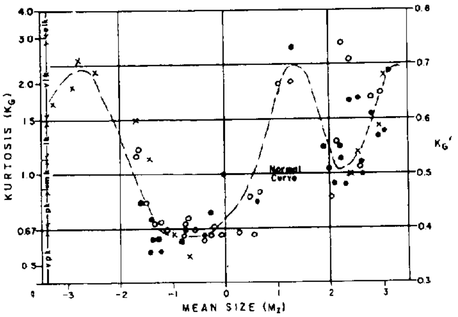

FIG. 13.—Scatter plot of kurtosis versus mean size. Spot samples shown by filled circles, channel samples by open circles, and special samples by X. The trend is complex. Letters along the left margin give verbal limits on kurtosis; vpk, very platykurtic; pk, platykurtic; mk, mesokurtic; lk, leptokurtic; vlk, very leptokurtic; elk, extremely leptokurtic. Points are actually plotted using the transformation $K_{G'}$ shown along the right margin; equivalent K_G is shown at left.

ous proportions. Not enough samples were collected to cover this theoretical skeleton with the flesh of numerous analyses. Again, the pure sand mode and the pure gravel mode by themselves give nearly normal curves with $K_G = 1.00$. The addition of very small amounts (3 to 10 percent) of another mode means that the sorting in the tails is worsened while the sorting in the central part remains good; hence the curves become strongly leptokurtic with K_G considerably higher than 1.00. Further additions of the new mode give rise to a strongly bimodal sediment, and if the two modes are subequal (in proportions anywhere between 25:75 to 75:25), then the sediment becomes very platykurtic. When the second mode attains 90 percent or more of the sediment, the curve once more becomes leptokurtic, and when the second mode reaches 100 percent, a normal curve with $K_G = 1.00$ should occur again.

This theoretical trend is well illustrated in Brazos bar samples. The largest clustering of values is around $K_G = .60 - .65$ (very platykurtic) at $M_z = -0.8\phi$. These represent the sandy gravels with subequal amounts of the two modes, i.e. 45 to 70 percent gravel. The leptokurtic curves when M_z becomes finer than 2.5ϕ indicate again the addition of a small tail of silt to the sand mode.

Standard Deviation versus Skewness.—It is evident that if standard deviation is a function of mean size, and if skewness is also a function of mean size, then sorting and skewness will bear a mathematical relation to each other. However, the exact nature of their relation was a great surprise: the two variables form a scatter trend in the form of a nearly circular ring (fig. 14)! The reason becomes obvious on a moment's thought, however; symmetrical curves may be obtained either in (1) unimodal samples with good sorting, or (2) equal mixtures of the two modes which have the poorest possible sorting for this suite of samples. The most extreme values of skewness will be shown by samples with one mode dominant and the other subordinate. These show moderate sorting, and the skewness can be either positive or negative. If one starts with the finest Brazos bar samples (the silty sands) and progresses through coarser and coarser samples to the pure gravels, the loci of these points on the scatter plot is a clockwise circular or elliptical path starting at A and going through B to C.

Standard Deviation versus Kurtosis.—Worst sorting is found in the bimodal mixtures with equal amounts of the two modes, and these also have lowest kurtosis (fig. 15). Highest kurtosis is found in those samples with one mode dominant and the other very subordinate; these have moderate sorting. Unimodal sediments produce normal kurtosis and are best sorted. Starting with the fin-

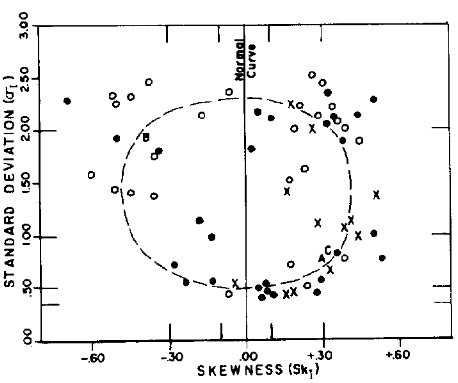

FIG. 14.—Scatter plot of skewness versus standard deviation. The trend is nearly circular. In order of increasing size, samples pass on this diagram clockwise from A through B to C.

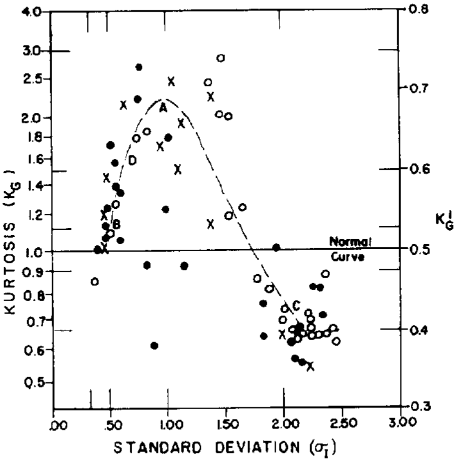

FIG. 15.—Scatter plot of kurtosis versus standard deviation. In order of increasing size, samples follow a regular progression on this diagram, from silty sands (A) to pure sands (B) to slightly gravelly sands (A again), to subequal mixtures of sand and gravel (C). As the gravel content increases, the changes are gone through in reverse, first travelling back to A (gravels with a little sand) and ending at D (nearly pure gravels). A sample of pure gravel would plot approximately at B, just as the samples of pure sand. If one mode is dominant and the other very subordinate, analyses plot at A; if both modes are nearly equal, the analysis plots at C.

est Brazos bar silty sands and going to coarser and coarser sediments, the path progressed on this diagram is a complicated inverted double V, from A through B, A, C, A, and D.

Skewness versus Kurtosis.—Both of these properties depend on the proportions of the two modes present. Thus a regular path is followed on the skewness versus kurtosis diagram as these proportions change and the mean size changes, shown in fig. 16 and 17. Starting with the nearly pure gravel mode[7] (field A), as more and more sand is added the points follow a U-shaped path through the platykurtic regions (modes now equal), then sweep up to reach highly leptokurtic and negatively skewed values in the sands with 7 to 20 percent gravel. As the gravel mode disappears, the path returns to the center point so that the pure sand mode gives a normal curve. But now, as the sediment continues to become finer by the addition of increasing quantities of the silt mode, the path moves again into positively skewed, leptokurtic regions, and if one continued to add silt, the entire cycle would presumably be repeated so that the path of the sand-silt mixtures would follow the path of the gravel-sand mixtures.

It will be noted that in the samples studied only very few analyses fall in the range of what would be considered "normal" curves, and the departures from normality are very great. This is because the two main modes on the Brazos bar are far apart and quite distinct. Sediments with two modes closer together or more poorly differentiated would give more nearly "normal" values of skewness and kurtosis, and all the points would be clustered around the center of the diagram. The farther apart the modes, the more extreme the values of skewness and kurtosis.

There is a conspicuous lack of samples that are symmetrical with high kurtosis, i.e. there is a gap in the upper center of the plot although the rest of the diagram is well filled. The reason is obvious. A leptokurtic, symmetrical sample would have to have a steep, well-sorted central mode, and, to re-

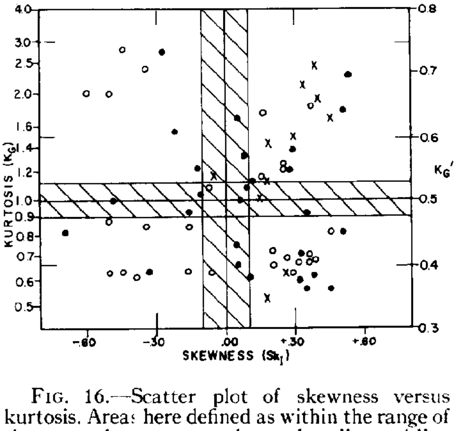

FIG. 16.—Scatter plot of skewness versus kurtosis. Areas here defined as within the range of the normal curve are shown by diagonal-line patterns; only 3 out of the 65 analyzed samples were normal with regard to both skewness and kurtosis. The wide range in skewness and kurtosis values is due to the wide separation between the modes and the ineffective sorting of the depositional environment.

[7] A sample of the gravel mode itself would give a nearly normal curve. All gravel samples collected in this study contained a little sand, hence were leptokurtic and positive-skewed.

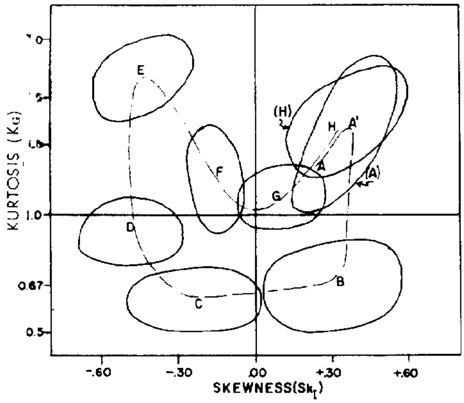

FIG. 17.—Skewness versus kurtosis, a summarization of figure 16 to show fields occupied by sediments of differing modal ratios. The dashed line extending from A through D to H shows the path followed by samples of continually decreasing grain size. Field A is occupied by the nearly pure gravels, with 80 to 95% gravel (perfectly pure gravel would plot as a normal curve). Fields B, C, and D are the sandy gravels containing respectively 55–80%, 37–55%, and 30–37% gravel. Between D and E is an unoccupied area representing the lack of samples with 20 to 30% gravel among the sediments analyzed. Field E, gravelly sand, contains 7–20% gravel, and field F contains 1–7%. G represents the nearly pure sands, with less than 1% gravel and less than 5% silt. Field H includes the silty sands, containing 5–15% silt. Fields H and A overlap because both consist of a dominant coarse mode and subordinate fine mode. If more silt were added, the sediments would follow the same path over a second cycle.

main symmetrical, must have a long "tail" of secondary modes equally balanced on both sides. In short, it must be a trimodal sediment with the middle mode most pronounced. Such sediments are understandably rare; only one Brazos bar sample fell in this area, and it *was* a trimodal mixture of dominant sand with small amounts of gravel and silt. Nearly all leptokurtic curves therefore show extreme skewness, either positive or negative.

Four-dimensional Plot.—In casting about for a way to depict all four parameters of the frequency distribution as one graph, it was discovered that the relation between mean, standard deviation and skewness (if plotted in three dimensions using x, z, and y axes respectively) formed a very close approximation to a helix (fig. 18). The clue that gave it away was the circular form of the standard deviation versus skewness trend, which would be obtained figuratively by looking down the rotational axis of the helix, i.e. looking down the x direction. The sinusoidal plots of mean versus standard deviation and mean versus skewness were simply side and top views of the horizontally-lying helix.

Figure 18 is an accurately plotted isometric projection of this three dimensional trend. The three sides of the "box" enclosing the helix represent two-dimensional projections of the trend, obtained by coplotting each pair of variables in turn. The fourth variable or dimension, kurtosis, is shown as pulsations of shading along the helix and the two-dimensional projections. We have constructed a large three-dimensional model of this trend to aid in visualizing the relationships. From a horizontal sheet of lucite (whose length represents the M_z scale, width the skewness scale) are hung plastic balls, one being placed at the proper coordinates for each sample. Height of the balls is determined by standard deviation. Kurtosis is shown by using balls of ten different colors in spectral order, to represent ten different ranges of kurtosis values.

Again, this helical trend shows how all four parameters of the frequency distribution are related to the relative abundance of the two modes. Best sorting and most "normal" curves occur if the sediment consists entirely of the sand mode. As the proportion of the gravel mode is increased, the size continually increases (moves left in the diagram), sorting continually worsens, and skewness goes to a maximum negative value and then decreases again. Kurtosis also attains a maximum (at a point just before maximum skewness), and then rapidly decreases passing quickly through the "normal" range to reach extreme platykurtic values when the 2 modes are present in equal amounts. As the gravel mode becomes dominant, the sequence of changes is undergone in reverse until, in the pure gravels, better-sorted normal curves again appear. The addition of silt, representing a third mode, starts a new cycle of the helix which is just beginning in the upward hook at the right of the trend.

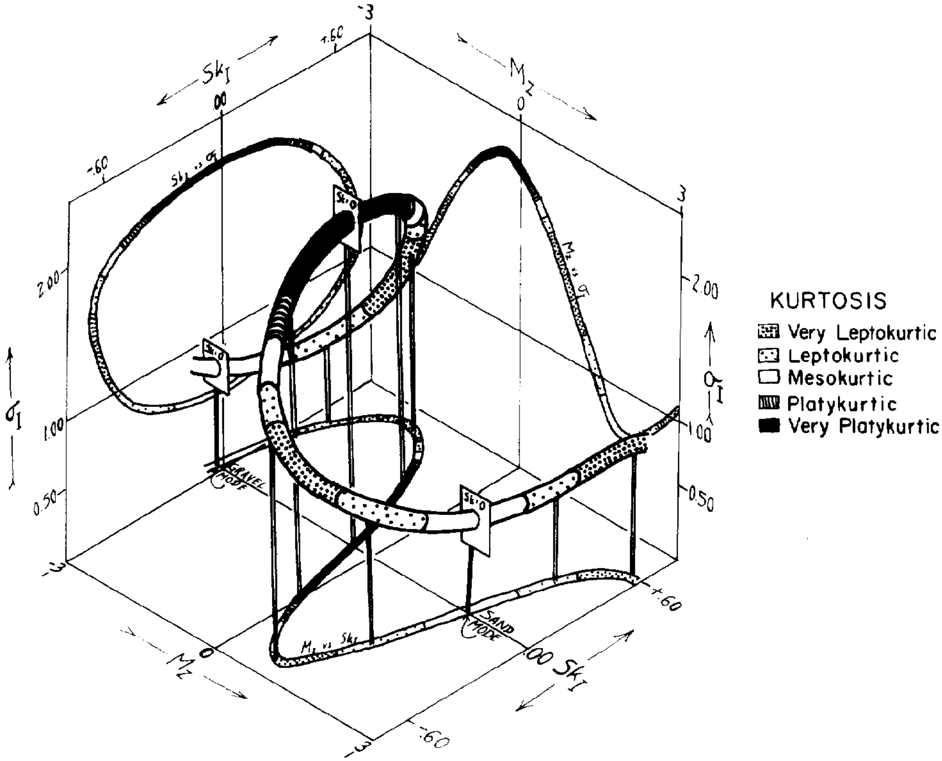

FIG. 18.—Four-variate graph, showing the relation between mean size (M_z), standard deviation (σ_I), skewness (Sk_I), and kurtosis. This is an accurately-plotted isometric projection of the helix which results when mean size, standard deviation and skewness are plotted against each other. Each of the three sides of the box containing the helix represents each pair of variables plotted in turn, hence correspond to two-dimensional projections of the helical trend. Standard deviation, the vertical dimension, is shown also by the height of the "supports" to the helix; points where the helix passes through the .00 skewness plane are shown by small "signboards." Kurtosis is shown by pulsations of shading along the helix and its three projections. The following limits are used: Very Platykurtic, K_G below 0.67; Platykurtic, K_G 0.67–0.90; Mesokurtic, K_G 0.90–1.11; Leptokurtic, K_G 1.11–1.50; and Very Leptokurtic, K_G 1.50–23.90.

The equation for the helix is

$$\begin{cases} \sigma_I = 1.5\phi - 0.75 \sin\left[60^\circ\,(M_z - 0.75\phi)\right] \pm 0.4\phi \\ Sk_I = -0.03\phi - 0.5 \sin\left[60^\circ\,(M_z + 0.75\phi)\right] \pm 0.12\phi \end{cases}$$

where σ_I and Sk_I represent the values for standard deviation and skewness (the dependent variables), M_z is the mean size in phi units, and the last term is the standard error of estimate (two-thirds of the values will fall within the predicted value plus or minus the standard error).

For example, consider a sediment with M_z of $+0.13\phi$. In the standard deviation equation, one finds the sine of $60^\circ\ (0.13-0.75) = \sin 60^\circ\ (-0.62) = \sin -37.2^\circ = -0.60$. When the rest of the equation is computed, the predicted value of σ_I is $1.5-0.75(-0.60) = 1.5+0.45 = 1.95\phi$, and two-thirds of the time the actual σ_I values will lie between 1.55ϕ and 2.35ϕ. In the skewness equation, one finds the sine of $60^\circ(0.13+0.75) = \sin 60^\circ(0.88) = \sin 52.8^\circ = 0.79$. The predicted value of Sk_I is then $0.50(0.79)-0.03-0.40-0.03 = -0.43$, and two-thirds of the actual values will lie between $-.31$ and $-.55$.

It is possible to work out a general equation for helical trends of this type, wherein the wave length of 360° gives the phi interval between modes (in this example the distance was about 6ϕ, therefore both the equations contained the factor $360°/6=60°$, and the skewness and standard deviation sine curves are one-quarter wave length out of phase (in this example $6\phi/4$ or 1.5ϕ, hence one equation contained the factor $M_z+0.75\phi$ and the other $M_z-0.75\phi$). The minimum point on the standard deviation sine curve coincides with the modal diameter, and the amplitude on the standard deviation curve is governed by the difference in σ_I between the average worst and the average best sorted samples. Actually it is not quite so simple, because seldom do both modes have equivalent σ_I values, but this type of formula may give a good approximation to the true quantities.

Preliminary work at the University of Texas and examination of previous published results (Inman, 1949) indicates that this helical trend applies in grain size distributions from many other environments. The helix probably goes through several more cycles, with each minimum of best sorting coinciding with a mode in the environment and each maximum coinciding with an inter-modal position, as shown in figure 19. These inter-modal regions can be easily identified by their platykurtic character.

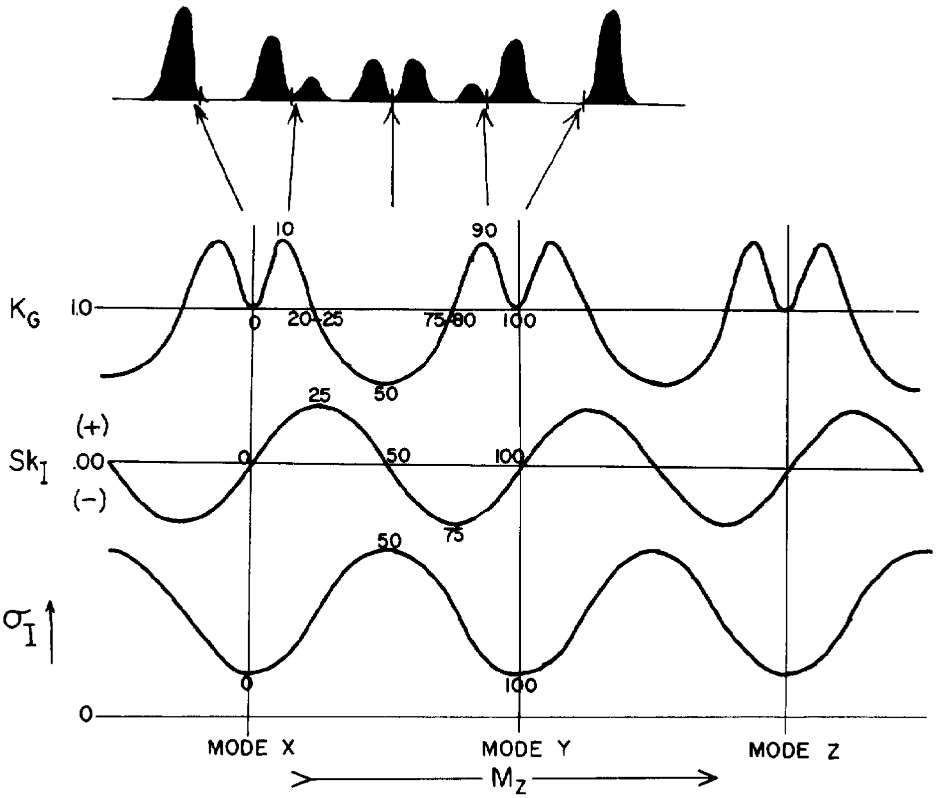

FIG. 19.—Theoretical variation of standard deviation (σ_I), skewness (Sk_I) and Kurtosis (K_G) as a function of mean size (M_Z) in a hypothetical polymodal sediment. Plots of σ_I and Sk_I form sine curves one-quarter wave length out of phase (actually combining to form a helix in three dimensions), while kurtosis forms a complex rhythmic curve. For mixtures of Mode X with Mode Y, the percentages given indicate the proportion of Mode Y present at critical points on each of the curves, provided the measures described herein are used. These percentages hold for the Brazos bar, but probably are slightly different in other environments. Shaded frequency curves at the top of the diagram illustrate the appearance of grain size curves for the critical points designated.

Work in progress indicates that in some neritic environments the modes are sand (or coarse silt) plus clay, also linked by a helix which would add another cycle to the right of the one shown here for the Brazos bar. Many sedimentary environments show only a segment of this helix; to have a complete cycle one needs two distinct and fairly widely separated modes, and within the suite of samples analyzed the pure end members as well as all intermediate mixtures must be present. If, for example, the samples examined consist only of the gradation from sand to clayey sand (say at most 35 percent clay), then only one-third (120°) of a helical cycle will be completed.

Special conditions may alter the form and position of the helix. Work in progress by Todd (1956) on some Eocene sands in Texas shows that these are polymodal sediments in which there usually is a small amount of clay, regardless of the size of the sand. This has the effect of making all the samples positive-skewed and leptokurtic, but here the addition of clay has simply shifted the axis of the helix in the xz plane; instead of being parallel to x with Sk_I equal nearly to .00 (giving nearly equal frequency of positive and negative skewness values), the axis is still in the xz plane but has a Sk_I value of .00 at 1ϕ and +.30 at 3ϕ. Similarly, Miller (1955), found in the Permian Pierce Canyon siltstone of southeast New Mexico a 180° segment of a helical trend but with the axis shifted back into positive Sk_I values because of the constant presence of small "tail" of clay. These positional shifts of the helical axis also exert a strong effect on kurtosis values, tending to shift them into leptokurtic regions.

Will these helical trends show up if other measures of grain size, such as those proposed by Inman, are used? The answer appears to be affirmative, but the trends are not as distinct. The measures proposed here are based on more points, hence are more sensitive and should be expected to give a better trend. For example, addition of a secondary mode affects σ_I and Sk_I if as little as 5 percent occurs, but 16 percent is required to affect Inman's $\sigma\phi$ or $\alpha\phi$. A sample with 30 percent gravel and 70 percent sand has nearly the same mean size as one with 70 percent gravel and 30 percent sand, using Inman's measures, but M_z is considerably more sensitive because the median is included in the calculation.

POSSIBLE GEOLOGICAL SIGNIFICANCE OF SKEWNESS AND KURTOSIS

If one may be permitted to extrapolate from a small study such as this one and enter the seductive field of generalization, it appears that both skewness and kurtosis are vital clues to the bimodality of a distribution, even when the modes are not immediately apparent. For example, these modes may be hidden in an obscure sidewise kick or gentle curvature of the cumulative plot on probability paper, enough to show up as non-normal skewness and kurtosis values, but not enough to show up as a secondary mode on a frequency curve or histogram. Strictly unimodal sediments (like some beach sands) should give normal curves; non-normal values of skewness and kurtosis indicate something "wrong" with the sediment, and indicate a mixing of two or more modal fractions. As an illustration of this principle, many dune sands on the South Texas coast have slight positive skewness and leptokurtosis, caused by the presence of a very minor coarse silt mode in a size finer than the principle mode.

Extreme high or low values of kurtosis imply that part of the sediment achieved its sorting elsewhere in a high-energy environment, and that it was transported essentially with its size characteristics unmodified into another environment where it was mixed with another type of material. The new environment is one of less effective sorting energy so that the two distributions retain their individual characteristics—i.e. the mixed sediment is strongly bimodal. In the Brazos bar, extreme kurtosis values are attained because the bulk of the sand apparently received its sorting in the parent Cretaceous marine sediments, but is now being deposited in the less efficient sorting environment of a river bar, where it is rapidly dumped together with gravel or silt. In neritic sediments, extreme kurtosis values are common because the sand mode achieves good sorting in the high-energy environment of the beach, and then is transported en masse by storms to the neritic environment, where it becomes mixed with clay and hence is finally deposited in a medium of low sorting efficiency. If the sediments are near the source of the sand, they are characteristically leptokurtic and positive-skewed because the sand is in excess. The more extreme the kurtosis values, the more extreme is the sorting of the modes in their previous environment and the less effective is the sorting in the present environment. Thus one may conclude that kurtosis and skewness are very valuable clues to the "genealogy" of a sediment.

CONCLUSIONS

Once a relationship is established in an ideal case, where the changes are laid out before the observer in their most perfect form, one soon learns to recognize the same relationships in less ideal examples, where the changes are obscure. The obscure examples, hitherto unfathomable, are explained in the light provided by the ideal examples. So it has been with the Brazos bar study. We have studied a simple environment, where the changes follow an orderly helical progression because of the ideally bimodal character. Now, fortified with the knowledge of the ideal trend, we have been able to unravel many once-puzzling relationships in other sedimentary suites of more complex nature and to understand better what is going on in the sedimentary environment. The meaning of skewness and kurtosis has, we feel, been ascertained: they are vitally important distinguishing characteristics of bimodal sediments and enable us to recognize bimodality where it was previously obscure. The changes of skewness, kurtosis, and sorting with sediment transport are probably simple functions of the ratio between the two modes of the sediment. The equations tying these variables together will, we hope, be of some value in distinguishing sedimentary environments. It is not the absolute values of parameters themselves, but their four-dimensional relationships to each other which offer the best hope of further progress.

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